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# 2 – Lenses, Magnification & Beam Expanders

# 2 – Lenses, Magnification & Beam Expanders

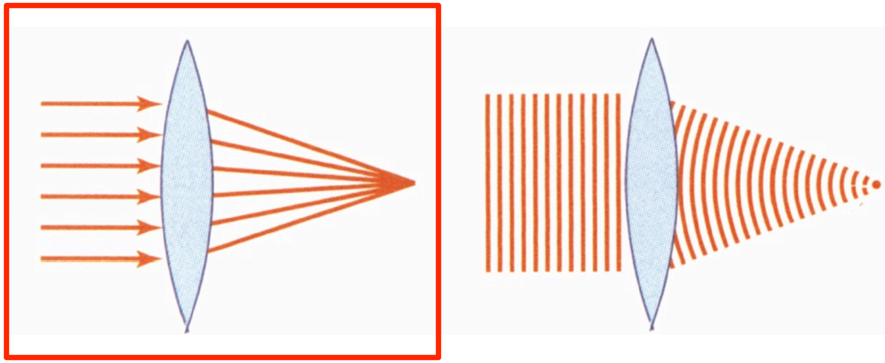
***Notice for quiz**: starting this week, you will have had used several of the equations we learned in lecture in the lab and lab homework, and therefore the quiz will also start to incorporate calculation style problems from the previous weeks lab.*

*I don't expect you to memorize the equations, you will always be able to look them up anyway…*

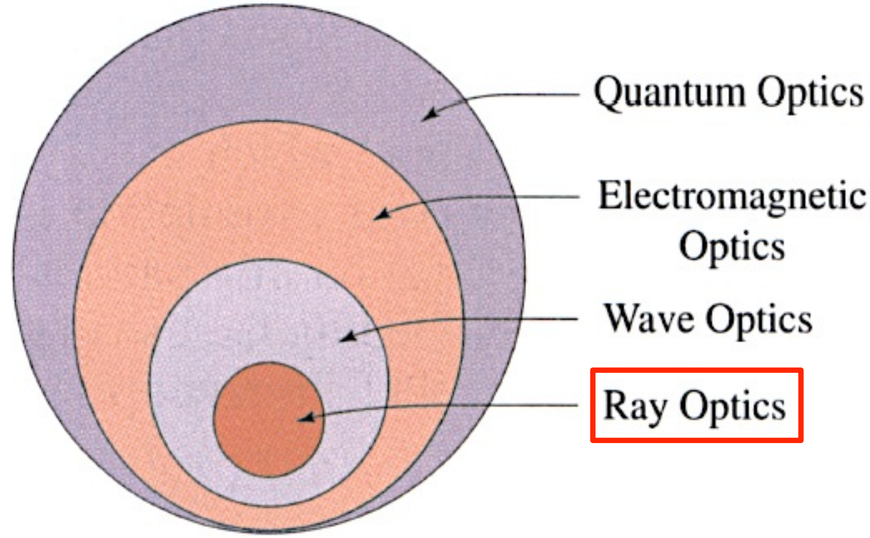
*Therefore each week you may bring to the quiz, 1/3rd of a sheet of paper with anything you want on it, anything!*

*You can keep adding to it, for example, such that 3 weeks from now it is a full sheet, and then you start a 2nd sheet, etc…*

- Today, we will only need to consider ray optics…



Credit: *Fund. Photonics – Fig. 2.3-1*




Credit: *Fund. Photonics – Fig. 1.0-1*

- Topics:

(1) derive the basic lens formula
(2) positive and negative lenses, and imaging planes
(3) multiple lenses in series (beam expanders, telescopes)
(4) advanced stuff (microscopes, numerical aperture, variable focus lenses)

*Figures today are mainly from CH1 of Fund. of Photonics or wiki.*

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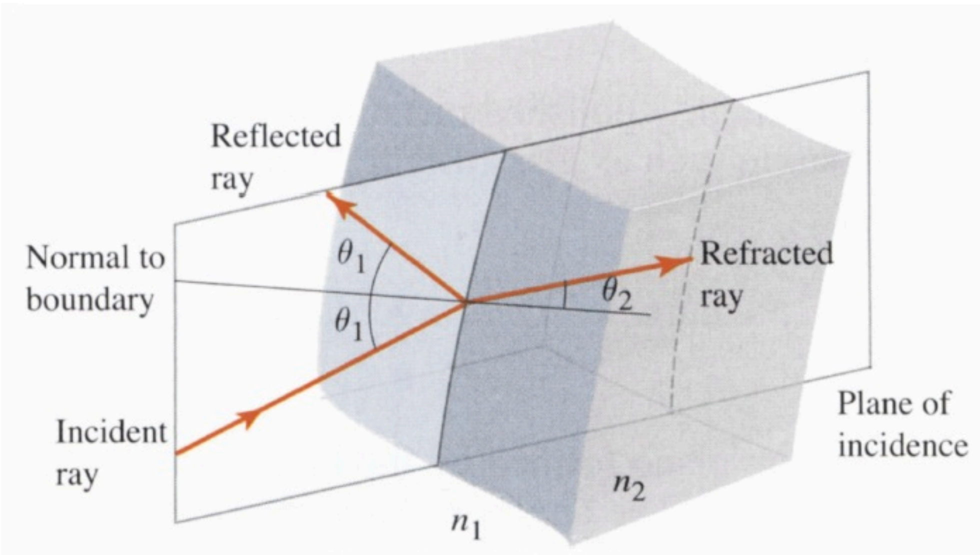
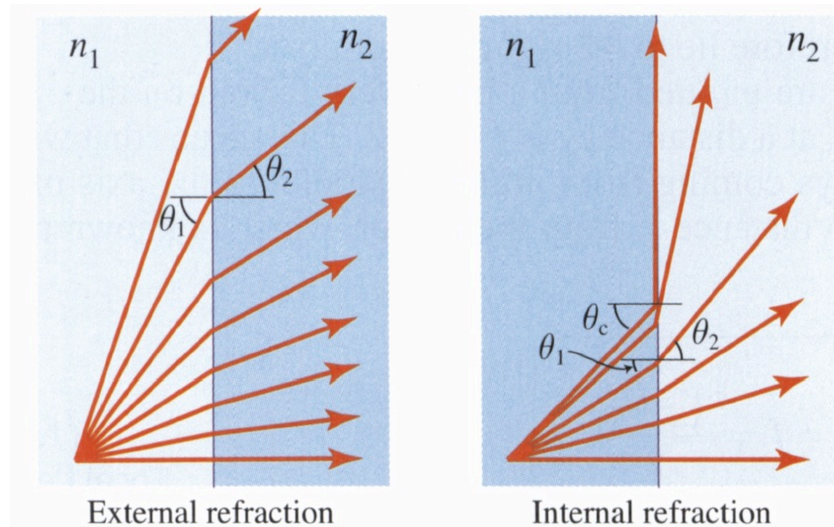
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Credit: Fund. Photonics



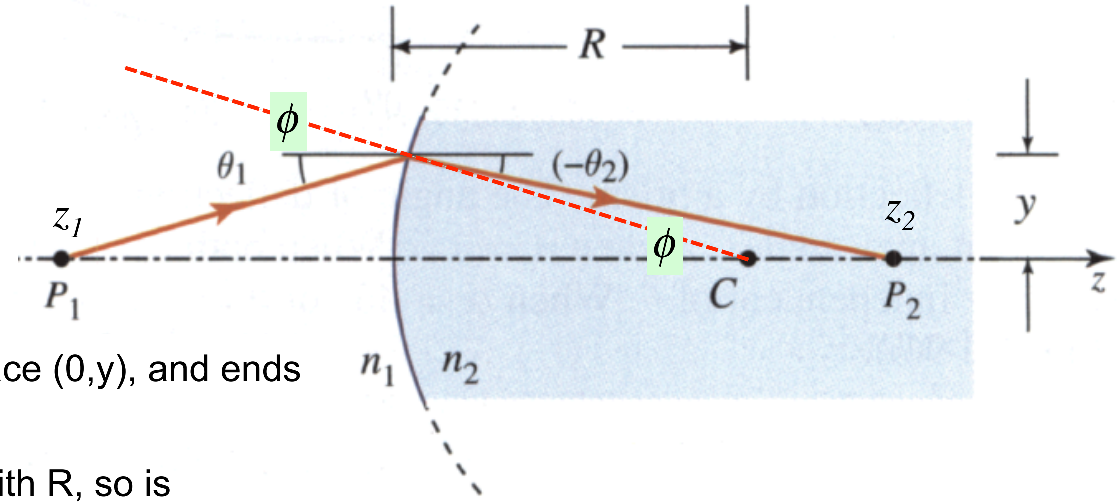




- Key point, before we go onto the next slide…

… we measure the incident AND the refracted angles both with respect to the SURFACE NORMAL.

- We have a spherical boundary with a radius of curvature R…
… by convention, $n_2 > n_1$ and the radius is positive in magnitude (convex).

*What if $n_2 < n_1$? By optics conventions, the radius would then be negative (concave!).*

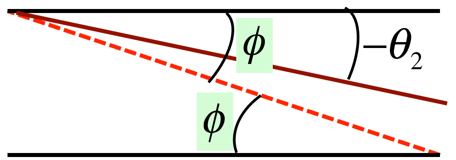



- Light travels from $P_1=(z_1,0)$, hits the surface (0,y), and ends up at $P_2=(z_2,0)$.

- Now add the dotted red line (is aligned with R, so is normal to surface, and gives us what we need to begin to use Snells law!

$$n_1 \sin(\theta_1 + \phi) = n_2 \sin(\phi - (-\theta_2))$$

*… don't worry about $-\theta_2$, it will work out…*




- For small angles, sin θ <u>*in rad*</u> approaches θ…

$$n_1(\theta_1 + \phi) = n_2(\phi - (-\theta_2))$$
$$\therefore \theta_2 = (n_1/n_2)\theta_1 + ((n_1 - n_2)/n_2)\,\phi$$

- Therefore $\theta_2 = (n_1/n_2)\theta_1 + ((n_1 - n_2)/n_2)\, y/R$

- Again, tan θ for small angles… $\theta_1 = y/z_1 \qquad -\theta_2 = y/z_2$

- And, for small angles, tan *Φ* approaches *Φ*… $\phi = y/R$

- Therefore

$$\frac{n_1}{z_1} + \frac{n_2}{z_2} = \frac{n_2 - n_1}{R}$$

- Bigger difference in refractive index, or smaller R, and the z's get smaller! Hmm…

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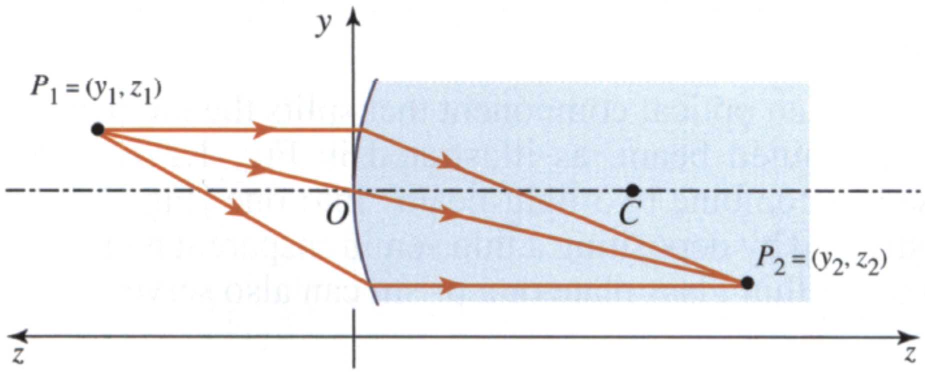
- Lets do the same thing now… but move $P_1$ to height $y_1$

- Consider light moving through the origin, for small angles is easy to see angles of incidence and refraction:

$$\theta_i = y_1 / z_1 \qquad \theta_r = -y_2 / z_2$$




- Relate via Snell's law: $y_2 = \frac{n_1}{n_2}\frac{z_2}{z_1} y_1$ ,and you can already see how changing z effects y (magnification)!

- Now apply to a thin lens in air (*thickness small compared to R, we will come back to this assumption later!*).

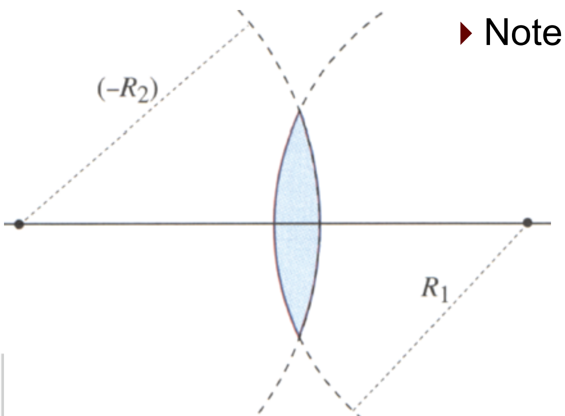
It can be shown that:

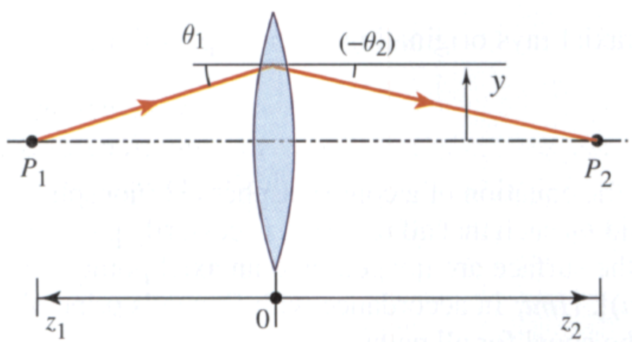
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right) \approx (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

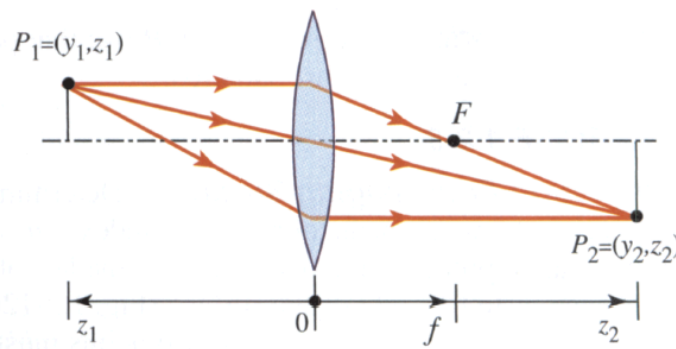
$$\frac{1}{f} = \frac{1}{z_1} + \frac{1}{z_2}$$

$$\frac{y_2}{y_1} = \frac{-z_2}{z_1} = M$$

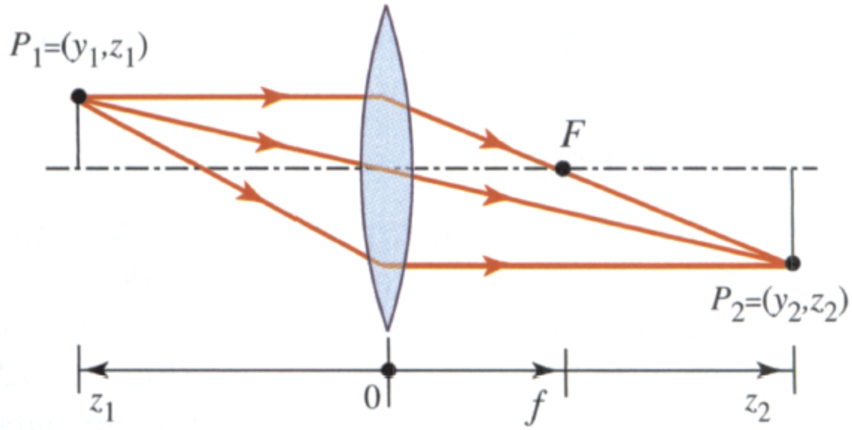
- Note, $z_2$ is positive (see vector). Also note, rays || to optical axis always focus to f !



- You can also easily derive the key relations using similar triangles (is a parallelogram, all sides proportional).




$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f}$$

$$\frac{y_2}{y_1}=\frac{-z_2}{z_1}=M$$

*If you wanted the image to be larger, not smaller, what would you change?*

- Lets do an example calculation for an 'object' at $P_1$ and an 'image' at $P_2$.

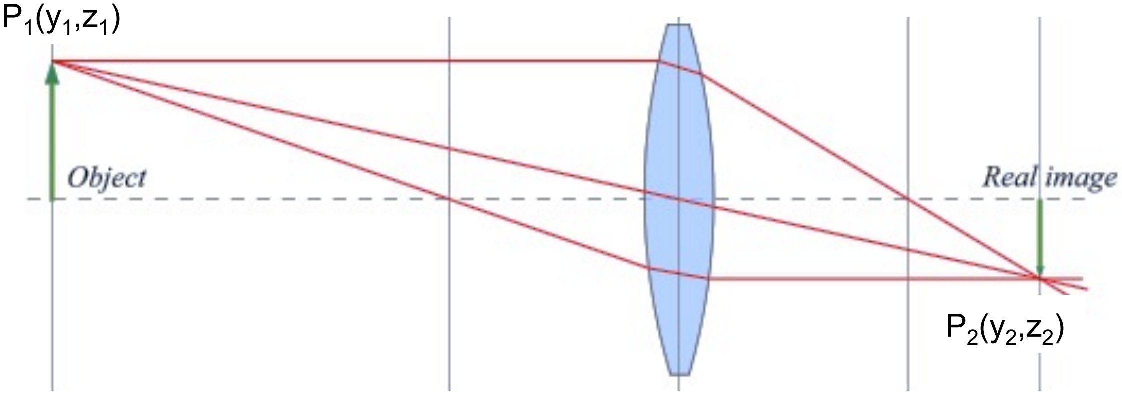
$$y_1=8cm \quad z_1=40cm \quad f=15cm$$

$$\frac{1}{40}+\frac{1}{z_2}=\frac{1}{15} \quad \therefore z_2=24cm$$

$$(positive, see\ vector)$$

$$\frac{y_2}{8}=\frac{-24}{40}=M \quad \therefore y_2=-4.8cm$$

$$\therefore M=-0.6$$




Note, the '-' in front of $z_2$ made sure that image was inverted in height $y_2$. The the magnification (M) also has – sign, which also just means that the height inverts (the magnitude is just a multiplier).

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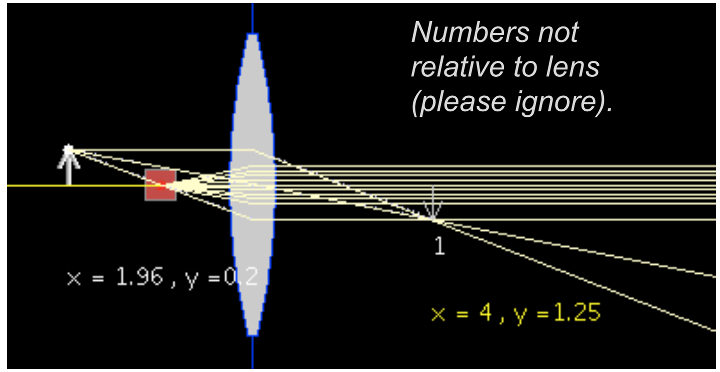
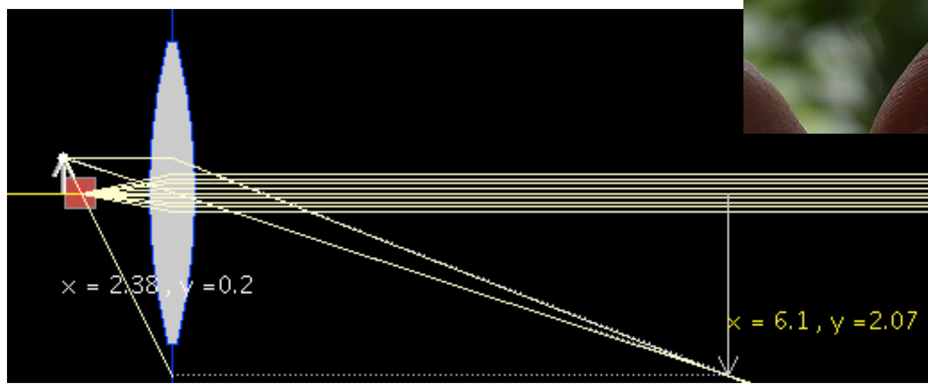
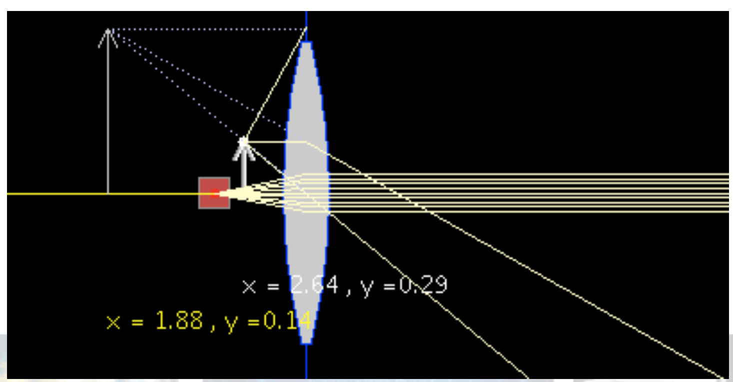
- Very helpful website: http://www.mtholyoke.edu/~mpeterso/classes/phys301/geomopti/lenses.html




- Notice in diagram at right increased magnification when you bring the object close to the focal point… notice in picture that object that is far away and the image shrinks instead!




- Careful! For a positive lens if you want to see a real image then the object must be BEYOND the focal length! We will talk more about 'virtual images' in a second.



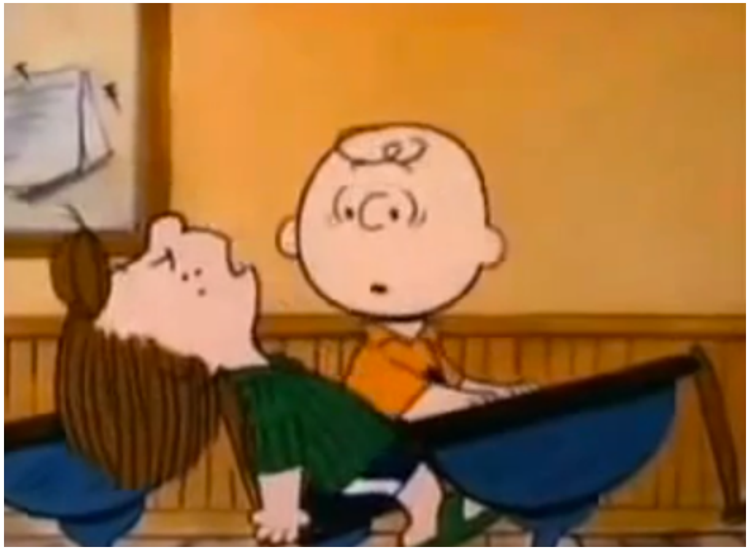
- For a positive lens,

(a) the image and object are always the same.
(b) the image and object are inverted.
(c) the image is always smaller than the object.
(d) none of the above.

- Parrallel rays of light incident on a postive lens:

(a) always focus down on the other side of the lens to the focal point of the lens.
(b) do not converge at any single point on the other side of the lens.
(c) always appear on the other side of the lens as parallel also.
(d) none of the above.

- Whew! That's enough. Lets take a break!



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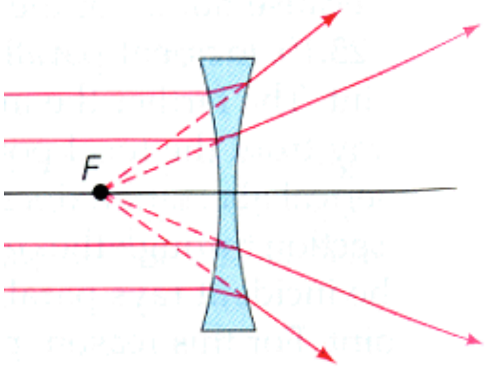
# Basic Negative (Diverging) Lens

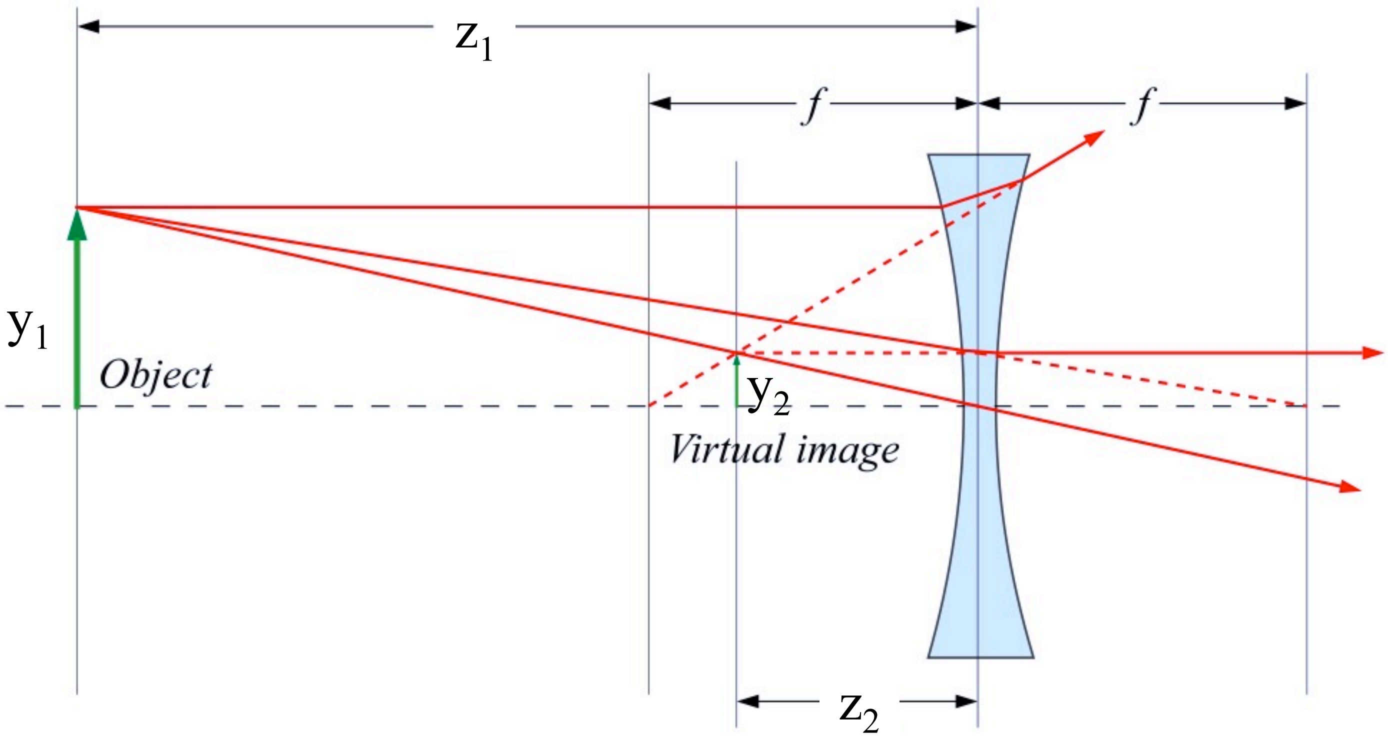
- Positive lenses are also called 'positive f', 'converging' or 'convex' lenses
- What would a 'diverging' lens look like in terms of shape (radius of curvature), ray path, and focal length? Trust the equations…

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

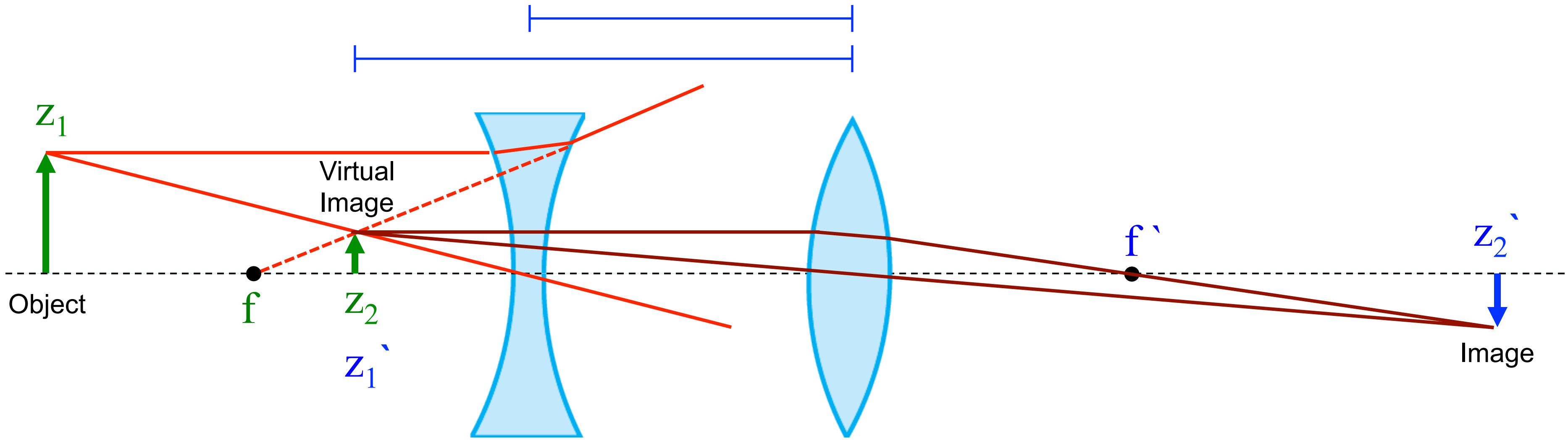
$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$\frac{y_2}{y_1} = \frac{-z_2}{z_1} = M$$




- Notice how negative R's result in a negative f.
- Notice that $z_2$ is now negative… (no longer on other side of lens), so our sign of magnification is what?
- Can we see the virtual image? Is there anything we can do to extract it and see it?




- This is one of the easiest ways to measure the focal length of a negative lens…

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$\frac{y_2}{y_1} = \frac{-z_2}{z_1} = M$$

- In lab this week, the object will be illuminated, and you will move a white card out past f` until you get a crisp image (which will be at $z_2$). Goal: figure out f for negative lens, how?

(1) You will know f` or the positive lens, so once you have $z_2$` for the positive lens you can calculate $z_1$` for the positive lens.

(2) Next, figure out $z_2$ for negative lens by subtracting $z_1$` for the positive lens from the distance between the center of both lenses (will give you a negative value, which is what you want!).

(3) Once you have $z_2$ for the negative lens, and you know $z_1$ for the negative lens, you can get f for the negative lens!

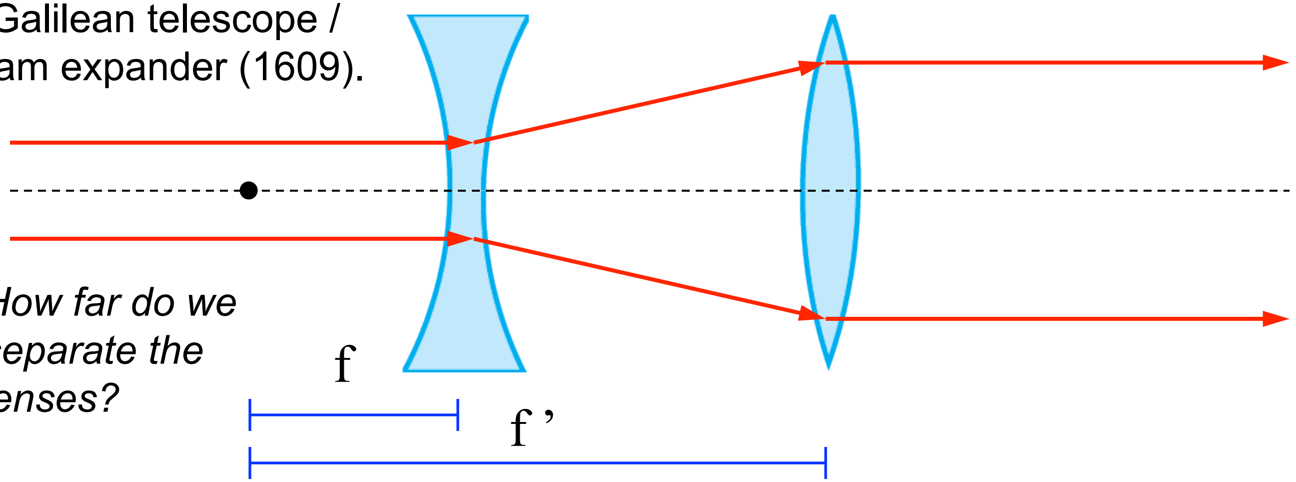
# Telescopes / Beam Expanders



- There are two major applications for the negative and positive lens combination (simple telescopes and beam expanders).

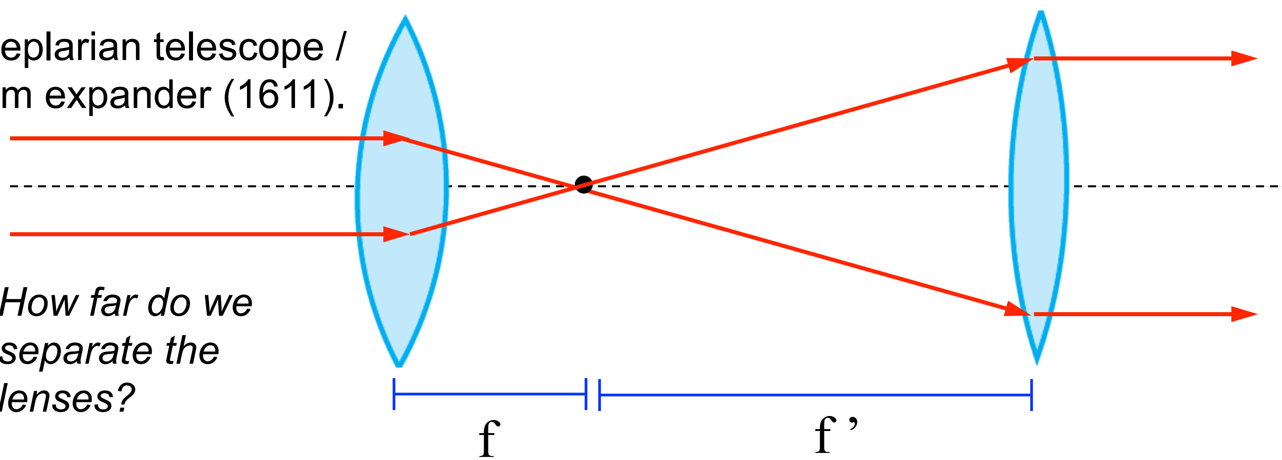
Left to right: beam expander.

- Galilean telescope / beam expander (1609).




*How far do we separate the lenses?*


- Keplarian telescope / beam expander (1611).




*How far do we separate the lenses?*

Right to left: telescope (tough to describe). Limited to 30X for Galilean...

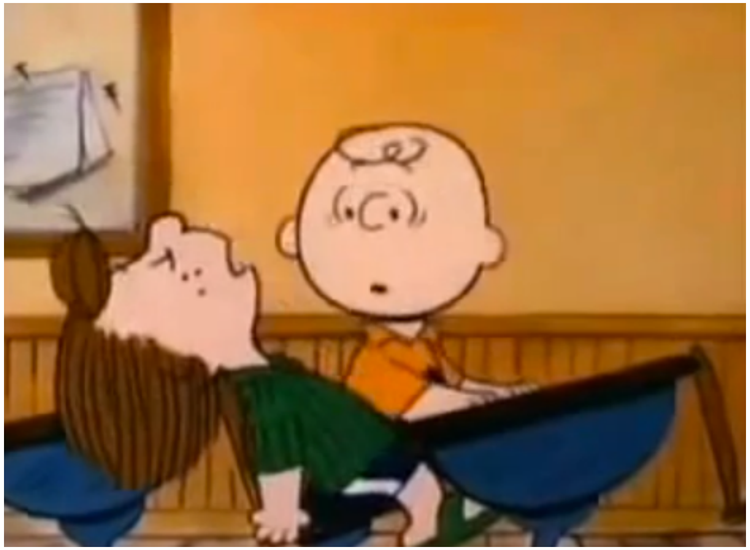
- For a single negative lens:

(a) there is only a real image.
(b) there is only a virtual image.
(c) both (a) and (b).
(d) neither (a) nor (b).

- A beam expander can be made of:

(a) a negative and positive lens spaced at the sum of the focal lengths for each lens.
(b) two positive lenses spaced at the sum of the focal lengths for each lens.
(c) both (a) and (b).
(d) neither (a) nor (b).

- Whew! That's enough. Lets take a break!



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- We will use beam expanders NUMEROUS times in this course, you will get good at assembling them!

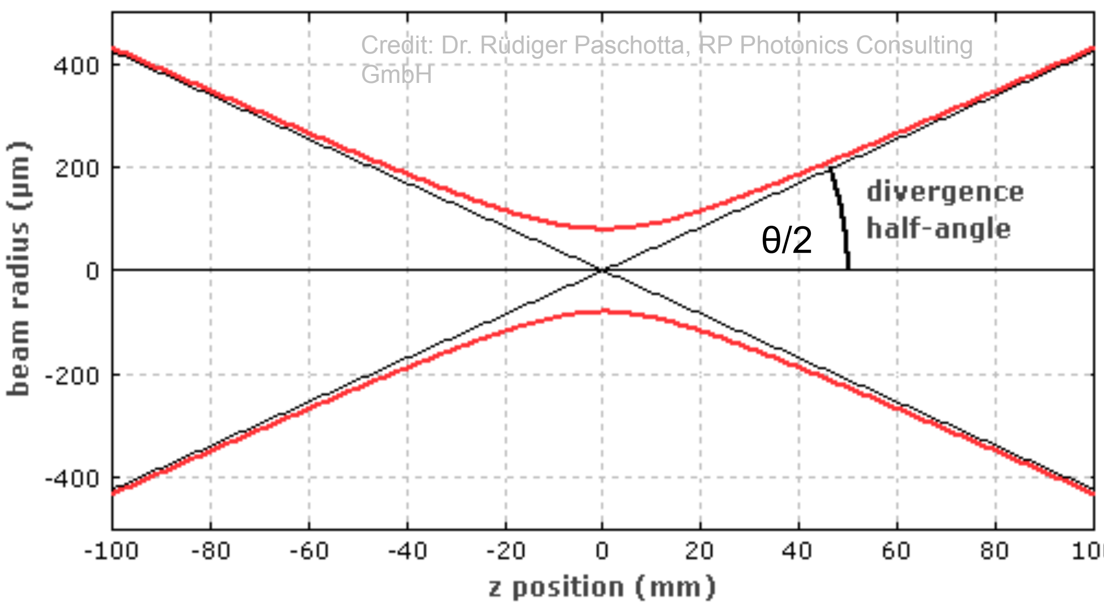
- Also, remember, you should be able to reverse the laser beam back through them, back into the laser, just like last week! You know you have a good setup once you achieve this!

- Another use for beam expanders is to reduce beam divergence (θ,radians)…

- First consider the smallest diameter ($d_0$) you can achieve for a beam (by focusing it), diffraction (next week's topic) will limit it to the following:

$$d_0\theta = \frac{4\lambda}{\pi}$$

The beam below is highly divergent, because they tried to make d only ~100-200 µm….




- So if the d θ product is a constant, then if we want a beam that is less divergent, then you must expand it!

$$d^2(z) = d_0^2 + \theta^2 z^2$$

- For our 633 nm laser, $d_0$ is ~ 1mm at the laser exit.

$$1x10^{-3}\theta = \frac{4x633x10^{-9}}{\pi}$$

$$\therefore \theta = 0.8mrad\ (\sim 0.05°)$$

- Laser size at the moon?

$$d^2(z) \approx (0.8x10^{-3})^2 x (3.8x10^8)^2$$

$$\therefore d = 304km!$$

- Expand to 1 m, and d at moon will be 1000X smaller (304 m).

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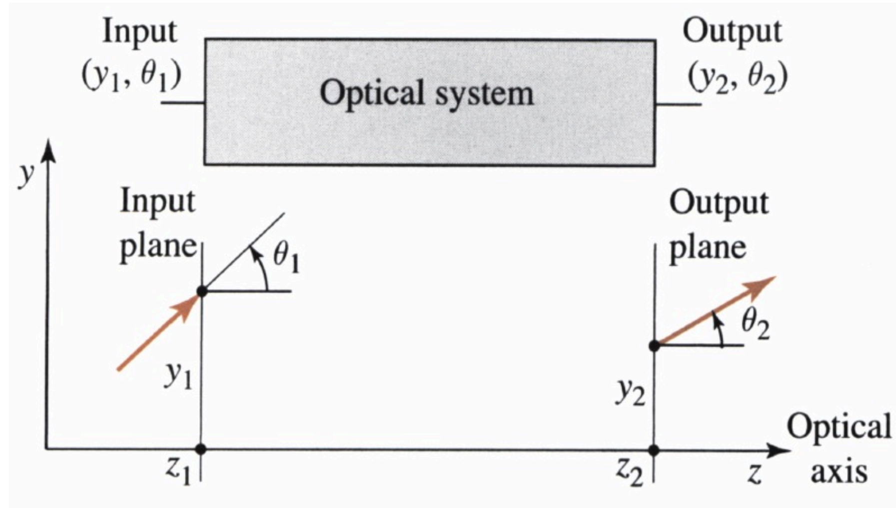
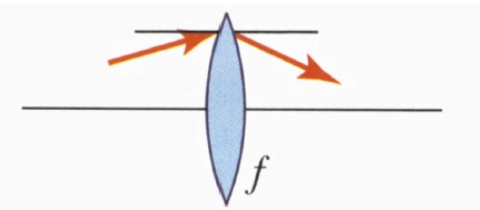
# Reminder: Ray Transfer Matrices

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

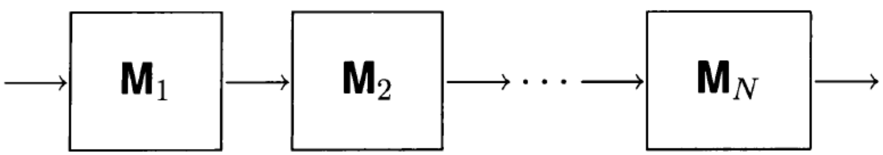



- Matrix for positive lens:

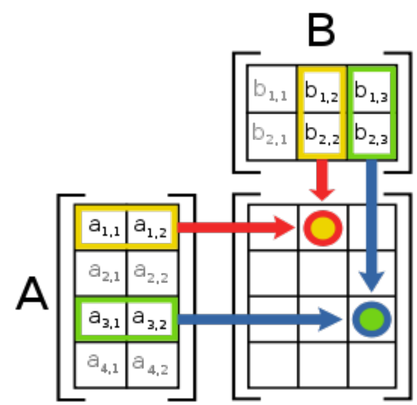



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

- Multiple optical elements? Just multiply the respective matrix representations for each optical element!




$$\mathbf{M} = \mathbf{M}_N \cdots \mathbf{M}_2 \mathbf{M}_1$$




- For a negative lens, do you just enter a negative f value into the matrix? You will figure it out for your homework this week … You will also verify your other experimental results.

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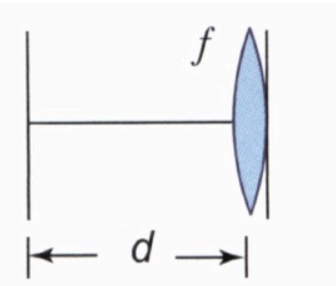
- Reminder, even the distance *d* from lens has a matrix! How about a lens of focal length *f* at a distance *d?*

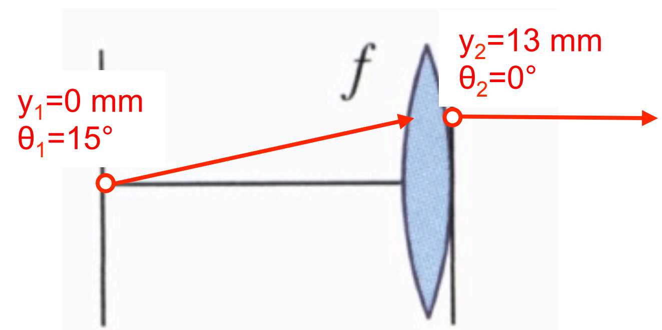



$$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \times \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & d+0 \\ -1/f+0 & -d/f+1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0-1/f & 1-d/f \end{bmatrix}$$

- What is $y_2,\theta_2$ if d= 50 mm, f= 50 mm, $y_1$=0 and $\theta_1$=15° (0.26 rad)? *(starting at focal point, so we know $\theta_2$ should be zero, right?).*

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0-1/f & 1-d/f \end{bmatrix} \times \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} y_1 + d\theta_1 \\ -y_1/f + \theta_1 - d\theta_1/f \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 50 \times 0.26 \\ 0 + 0.26 - 50 \times 0.26/50 \end{bmatrix} = \begin{bmatrix} 13\ mm \\ 0\ rad \end{bmatrix}$$

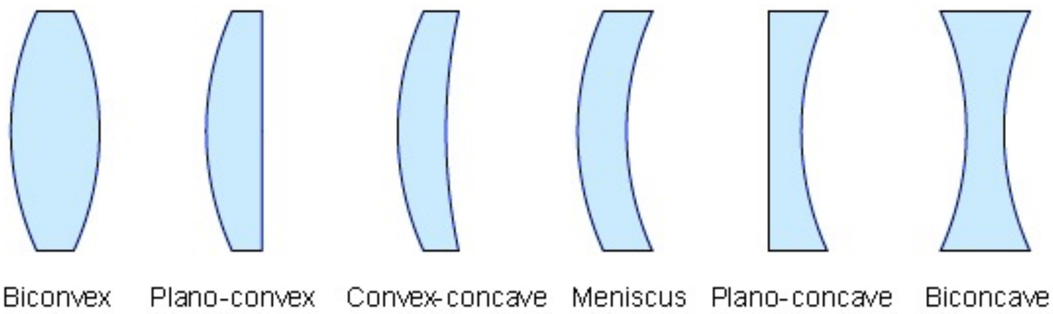


- There are numerous lens types… do we need to re-derive the equations? How do we calculate *f* for these?

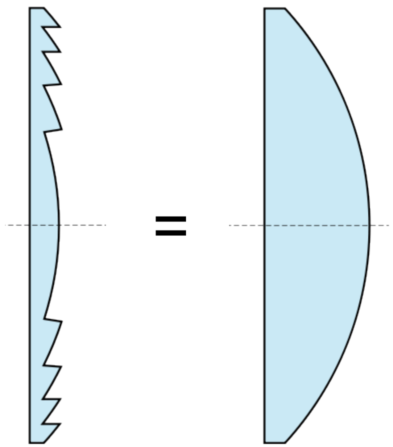
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$\frac{y_2}{y_1} = \frac{-z_2}{z_1} = M$$




- This is a Fresnel Lens which is valuable for large-area lenses (think about it), what is the key requirement to make it work? What is the main drawback?



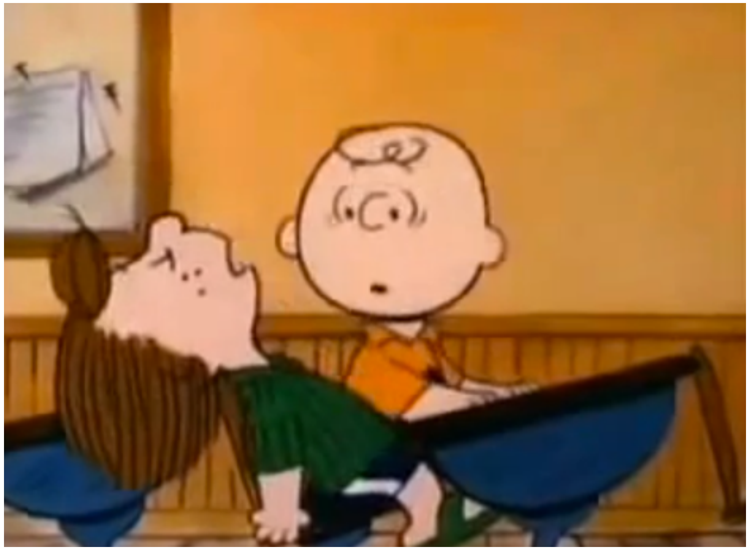

- The wider a laser beam is, the:

(a) more it diverges.
(b) less it diverges.
(c) no dependence on beam divergence.
(d) can't tell, need to know the type of laser.

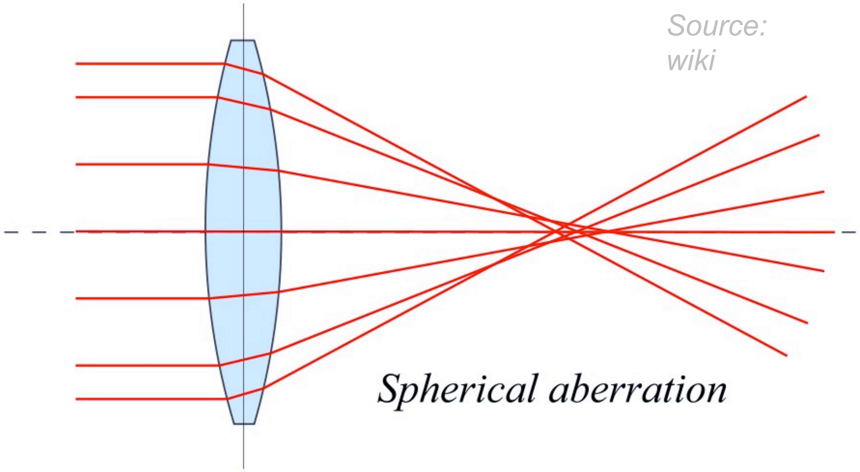
- Focusing light to a infinitely small spot is:

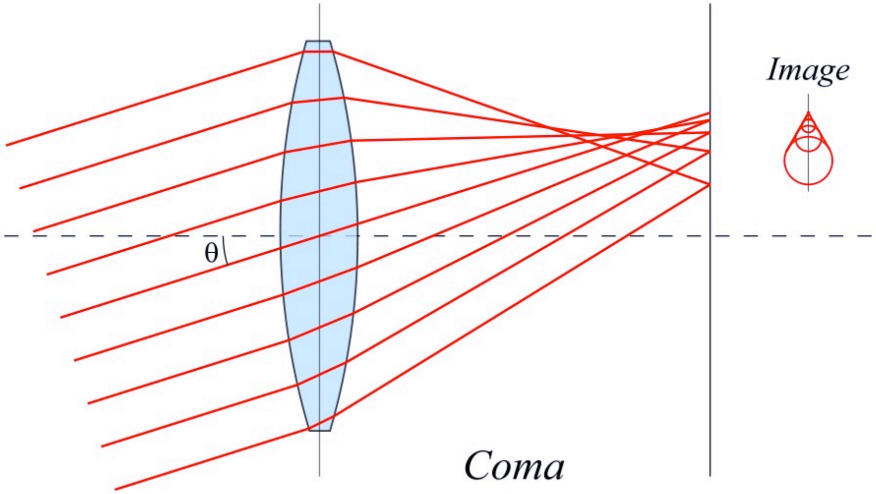
(a) possible.
(b) impossible.
(c) can't tell, need to know the type of light.
(d) I am too tired and confused to answer at this point…

- Whew! That's enough. Lets take a break!



- Be aware of non-ideal effects!




It is easiest to grind and polish lens with a spherical shape (think of the equipment), but it is not ideal and causes beams near the edges to miss the focal point… (will ask you why later).

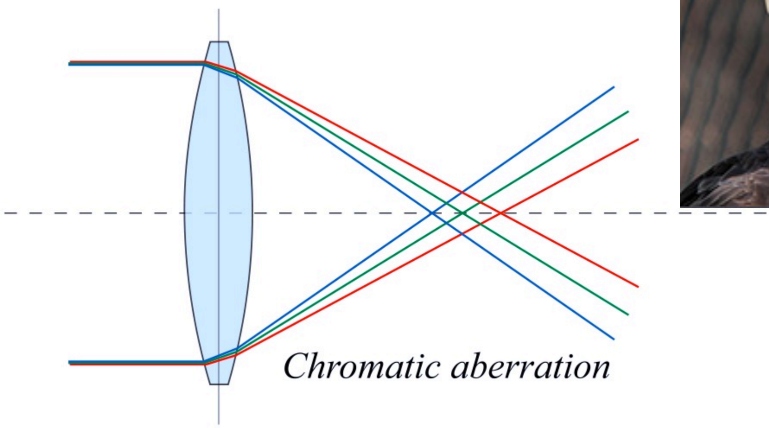
A helpful trick, for plano-convex is to have the convex side facing the beam source.

http://specialoptics.com/pdf/wp_bestform_laser_theory.pdf

Furthermore you want the incoming light well-aligned with the optical axis of the lens.

Not all applications can do this, though, so sometimes you again need a slighly modified lens…

- Be aware of non-ideal effects!

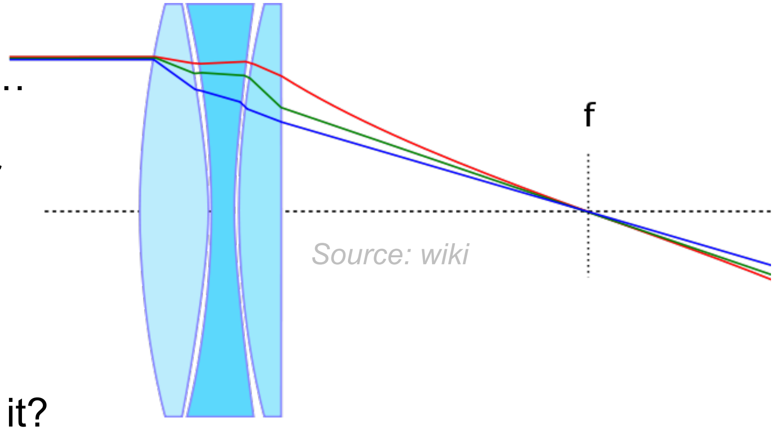




- Top photograph taken with a higher quality lens; bottom is taken with a wide angle lens showing visible chromatic aberration…

wiki/
File:Chromatic_aberration_(comparison).jpg

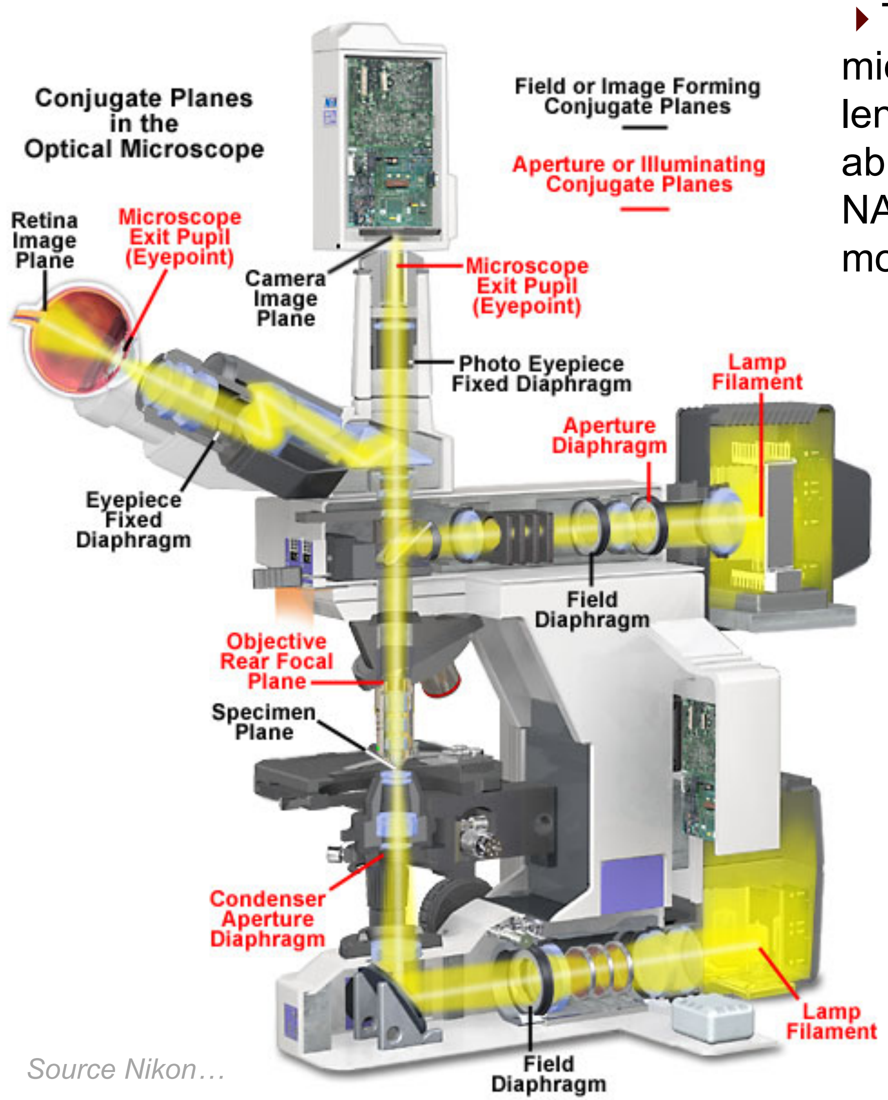
- What causes this? You should know the answer…
- Worse or better with high magnification/ zoom?
- How can we fix it? Any ideas?




Usually one element is made out of flint glass such as F2 with high dispersion, while the other is the opposite type lens (+/-R) and something like BK7 with low dispersion.

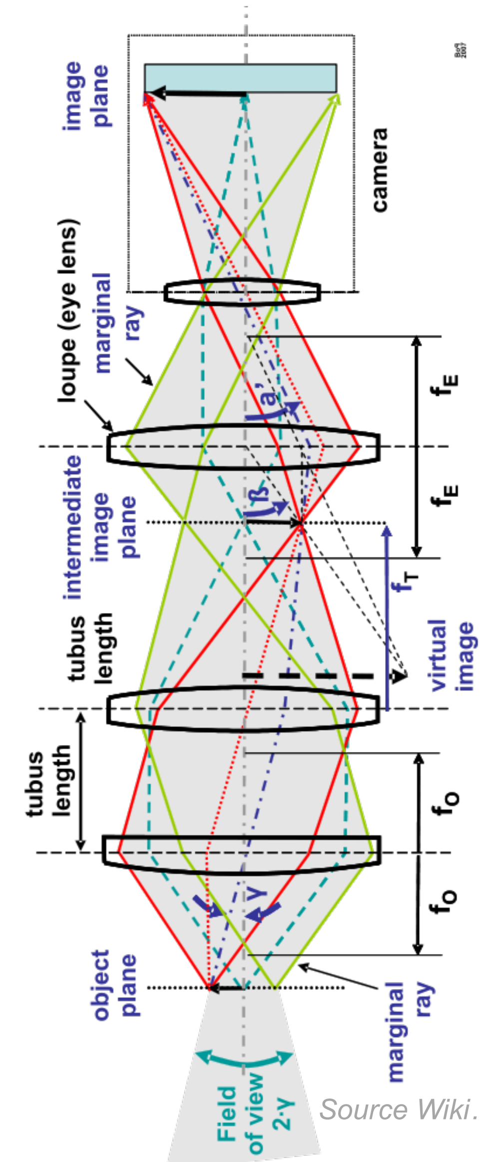
The elements are cemented together and shaped so that the chromatic aberration of one is counteracted by that of the other.

Drawback… the lens is weaker!



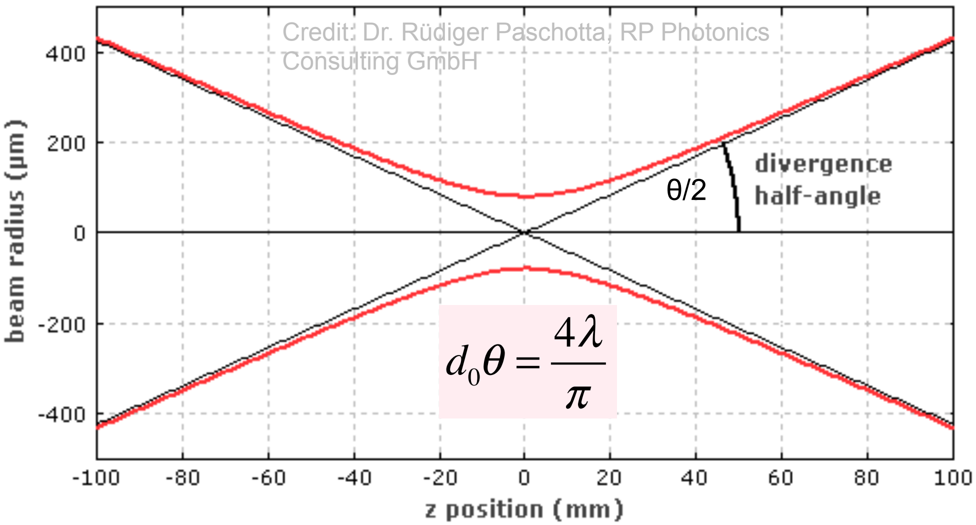

Source Nikon…

- Typically a compound microscope (multiple lenses to reduce chromatic abberation and allow high NA, more on NA in a moment…)




Source Wiki…

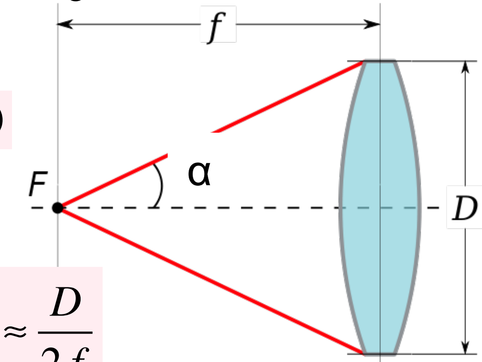
- Remember! One cannot make infinitely small laser spot due because of diffraction! (more next week…)
- Same limit for seeing something very small (but light moves in reverse)…




$$d_0\theta = \frac{4\lambda}{\pi}$$

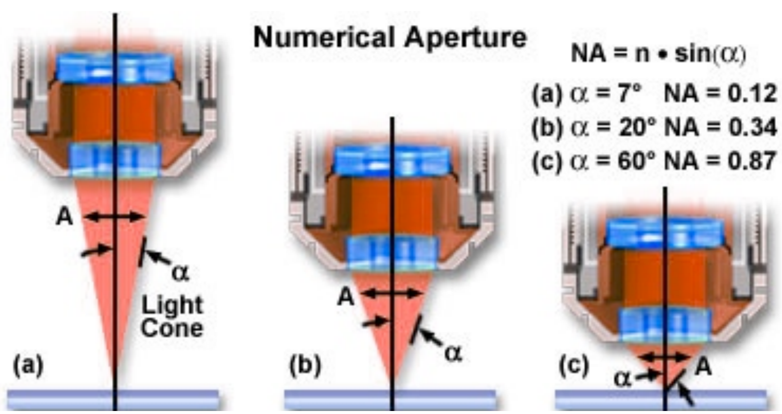
- If we want the smallest possible spot (e.g. magnify the smallest possible object) what is our only option?
- How would you design a lens to do this?

$$NA = n \times \sin(\alpha)$$




$$\alpha = \tan^{-1}\left(\frac{D}{2f}\right) \approx \frac{D}{2f}$$

- Example from Nikon (microscope objectives).




- Numerical aperture (NA) is focusing power (largest possible α) and also the light gathering power (also largest possible α). Higher NA, then lens gets closer! But what about the effect of *n*? …

- *n* is for the medium the lens is inside of! Sometimes an oil for larger NA and higher mag! *Eq. from slide 5, if both n's increase then R decreases!*

$$\frac{n_1}{z_1} + \frac{n_2}{z_2} = \frac{n_2 - n_1}{R}$$

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- Laser focusing and NA of course will impact optical data storage!

- Smaller wavelength of light, should also be able to image a smaller feature!

$$NA = n \times \sin(\alpha) \Leftarrow \alpha = \tan^{-1}\left(\frac{D}{2f}\right) \approx \frac{D}{2f}$$

$$\theta = 2\alpha \frac{\pi}{180}$$

$$d_0\theta = \frac{4\lambda}{\pi}$$

*Want both large θ and small λ!*

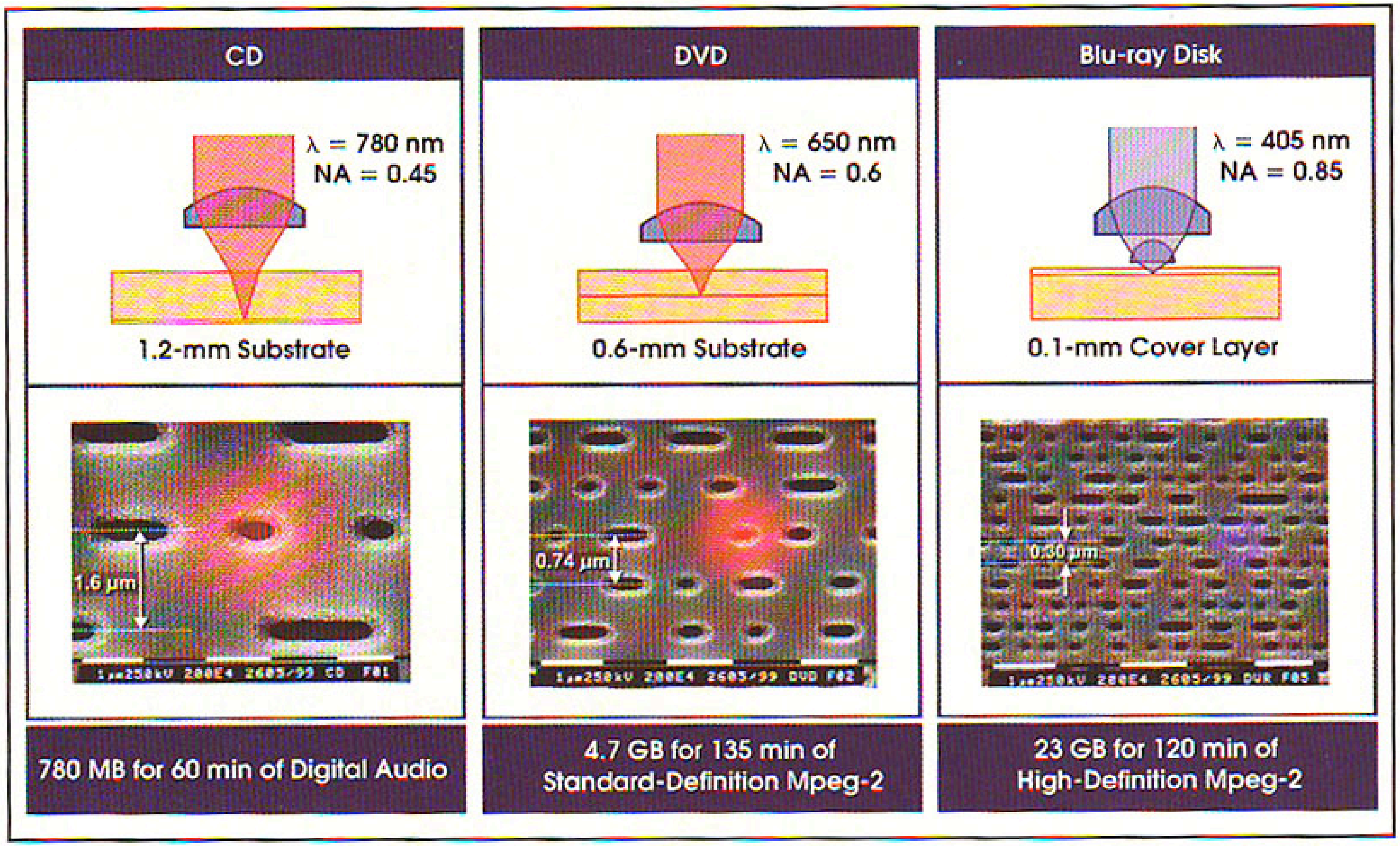



**Figure 5.** *Optical storage densities have increased significantly with the evolution of CD, DVD and Blu-ray technologies.*

*Photonics spectra..*

- Lenses should also allow you to collect light from a particular image plane, right?

- Everything not in that plane would not be in focus, but how to discard it? Use a pin-hole aperture at the focal point!

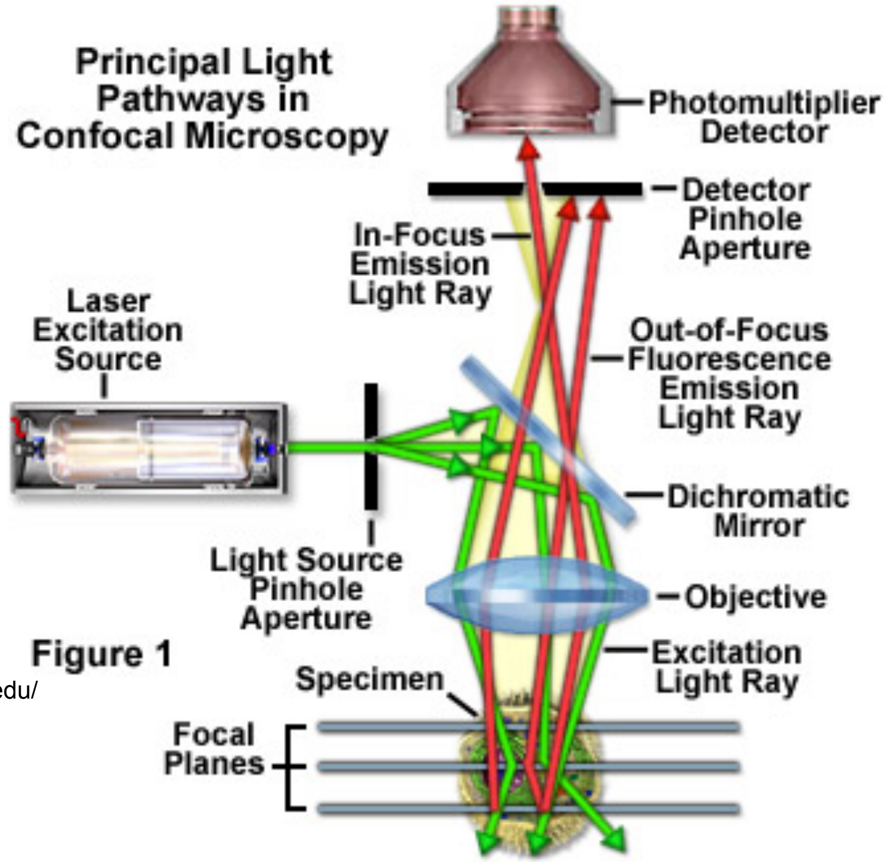
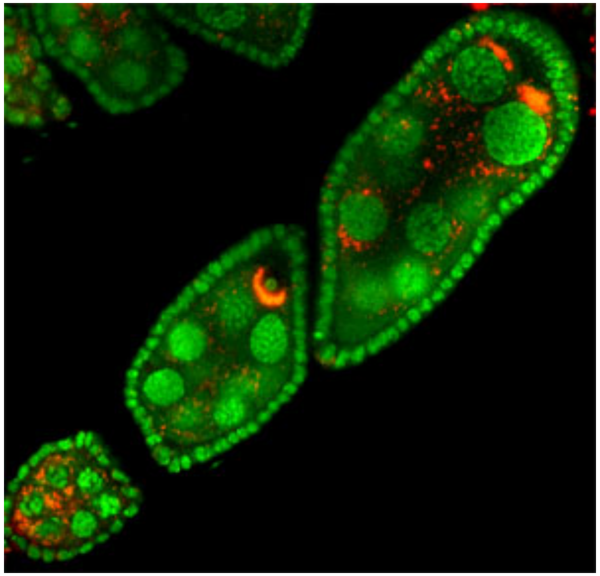



Figure 1

microbiology.ucsc.edu/confocal.html

- Take cross-section image from within a semi-transparent object that is fluorescing.



*The Sullivan laboratory uses confocal microscopy to examine at the cellular level the effects of the bacteria Wolbachia on reproductive mechanisms of the fruit fly D. melanogaster. In this image, DNA is labeled green, and Wolbachia are red.*

varioptic
a Business Unit of Parrot

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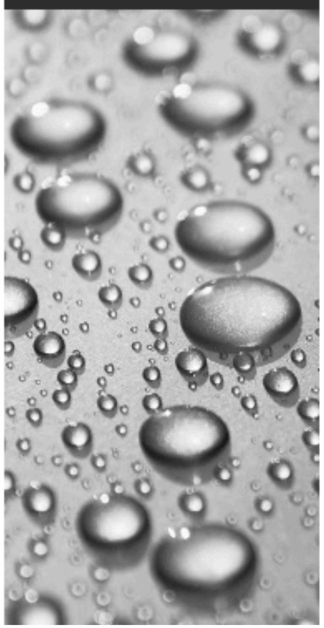

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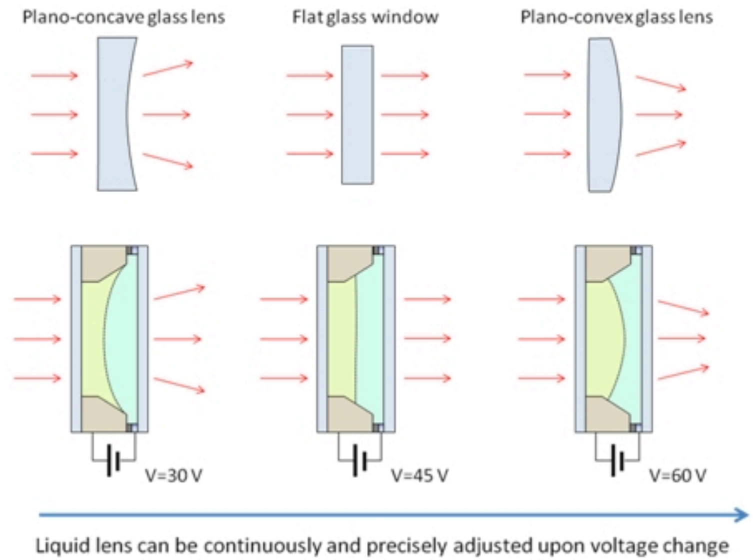
# ELECTROWETTING ›

For centuries scientists have combined lenses of very different optical powers, from diverging lenses to highly converging ones, to design and manufacture complex optical instruments. Until now the optical properties of all these lenses, either made from glass or plastic and typically requiring months to be produced, were always fixed. Varioptic has now created a « smart lens » that can be reconfigured on demand with just a variation of voltage. The lens can adapt rapidly and continuously from diverging to converging and be modeled to support all key optical functions, starting with Auto-Focus and Optical Image Stabilization.





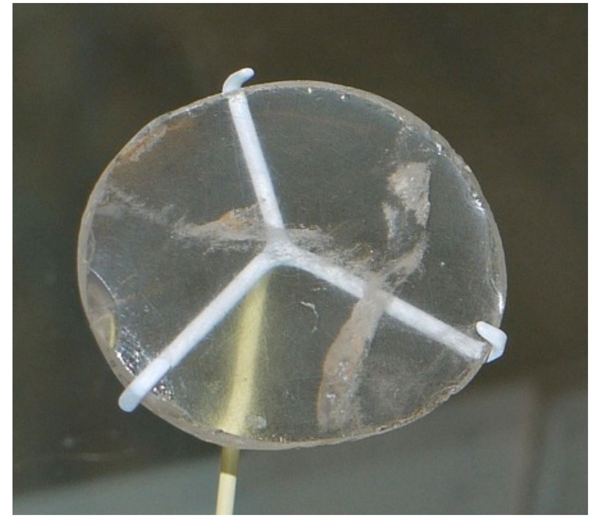
The technology uses the Electrowetting principle and a combination of transparent and optically defect-free liquids to create a lens and change its characteristics in real time. Liquids have been used since 40 years in optical systems for high-end products such as goggles or camcorders, but Varioptic's innovation is to have created a real-time programmable platform that offers to change the shape of the liquids in a very fast, repeatable, precise and controlled way.

- Numerical aperture for a lens is the:

(a) focusing power (largest possible α)
(b) light gathering power (also largest possible α)
(c) both (a) and (b).
(d) neither (a) nor (b).

- Fundamentally, lenses have difficulty simultaneously focusing light of all wavelengths because of:

(a) imperfections on the lens surface.
(b) refraction.
(c) dispersion.
(d) mone of the above.



- Last question: The oldest lens artifact is the Nimrud lens from ancient Assyria (>1000 yrs old,). What did they likely use it for? Think of what a kid would do…

- *Great imaging/microscope website with many tutorials and java applets: http:// micro.magnet.fsu.edu/primer/index.html*